Princess Sumaya University for Technology
Communication Engineering Department

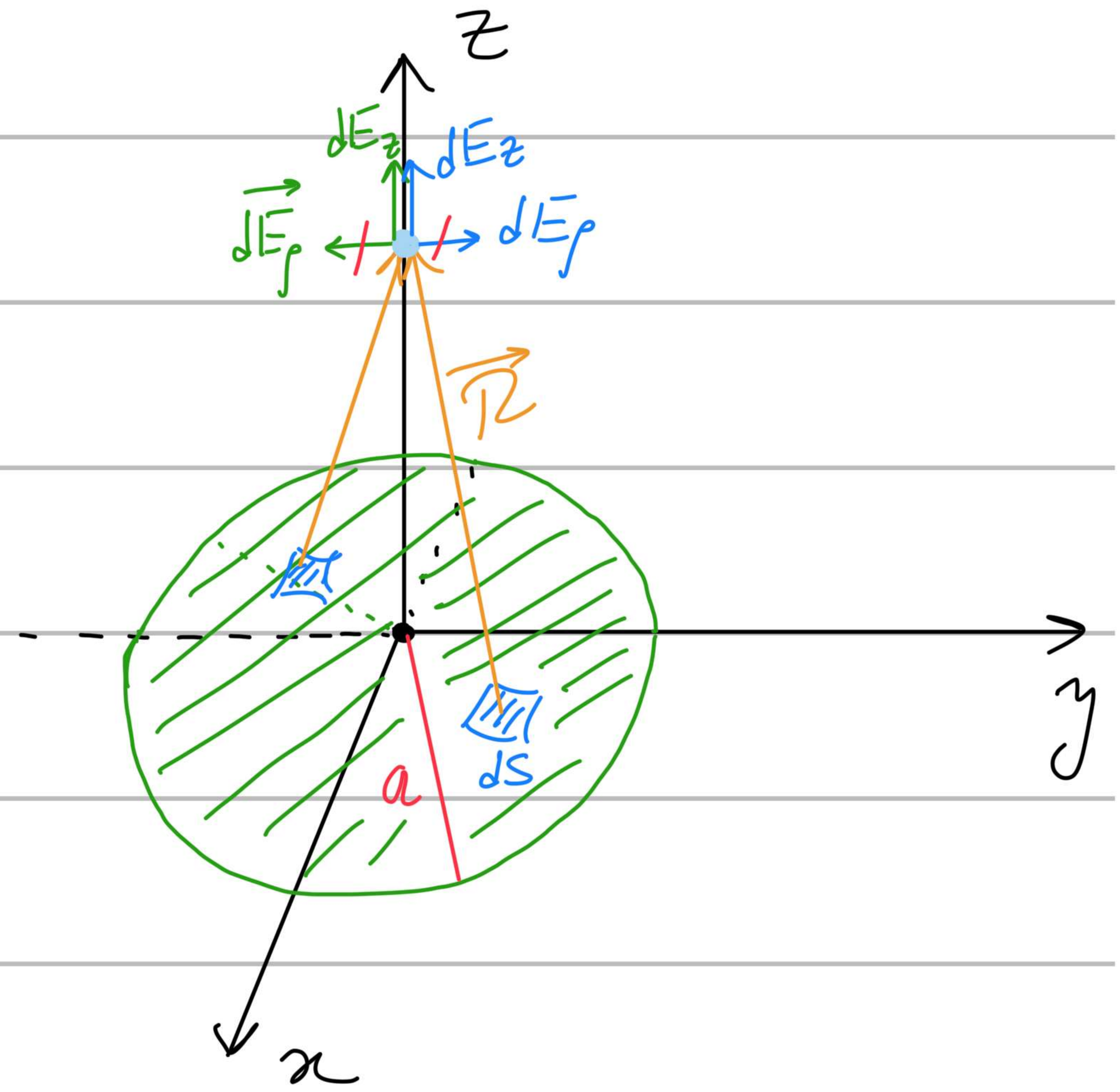
Electromagnetics I
Quiz 1

A circular disk of radius *a* is uniformly charged with ρ_S C/m^2. If the disk lies on the $z = 0$ plane with its axis along the z-axis, find $\boldsymbol{E}$ at point $P\,(0,\ 0,\ h)$.

Soln:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{|\vec{R}|^2}\vec{a_R}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_s\, dS\, \vec{R}}{|\vec{R}|^3}$$



But $\vec{R} = -\rho\vec{a_\rho} + h\vec{a_z}$

$$\therefore \vec{E} = \frac{\rho_s}{4\pi\varepsilon_0}\iint_S \frac{(-\rho\vec{a_\rho} + h\vec{a_z})}{(\rho^2+h^2)^{3/2}}\,dS$$

But due to symmetry, $\vec{a_\rho}$ adds up to zero. Also $dS = \rho\, d\rho\, d\phi$.

$$\therefore \vec{E} = \frac{\rho_s \cdot h}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{a} \frac{\rho\,\vec{a_z}}{(\rho^2+h^2)^{3/2}}\,d\rho\, d\phi = \frac{2\pi\rho_s h}{4\pi\varepsilon_0}\int_0^{a}\frac{\rho\,\vec{a_z}}{(\rho^2+h^2)^{3/2}}\,d\rho$$

Let $t = (\rho^2+h^2)^{3/2} \Rightarrow t^{2/3} = \rho^2 + h^2 \Rightarrow \frac{2}{3}t^{-1/3}dt = 2\rho\, d\rho$

$$\Rightarrow \frac{1}{3}t^{-1/3}dt = \rho\, d\rho$$

$$\therefore \vec{E} = \frac{\rho_s h}{2\varepsilon_0}\int_0^{a}\frac{1}{3}t^{-1/3}\cdot\frac{1}{t}\,dt\,\vec{a_z} = \frac{\rho_s h}{2\varepsilon_0}\int_0^{a}\frac{1}{3}t^{-4/3}\,dt\,\vec{a_z}$$

$$= \frac{\rho_s h}{2\varepsilon_0}\cdot\frac{1}{3}\cdot\frac{t^{-1/3}}{-1/3}\Bigg|_0^a \vec{a_z} = \frac{\rho_s h}{2\varepsilon_0}\left(\frac{-1}{\sqrt{\rho^2+h^2}}\Bigg|_0^a\right)\vec{a_z}$$

$$= \frac{\rho_s h}{2\varepsilon_0}\left(\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right)\vec{a_z} = \frac{\rho_s}{2\varepsilon_0}\left(1 - \frac{h}{\sqrt{a^2+h^2}}\right)\vec{a_z}$$